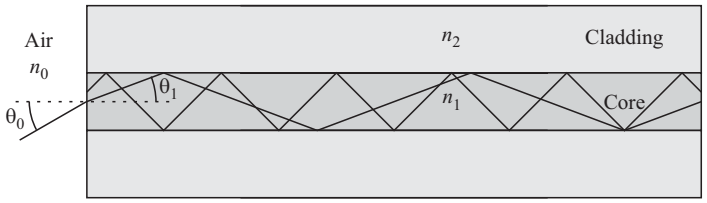


**Figure 2.5** Propagation of light rays in optical fiber by total internal reflection.

It can be shown using Snell's law (see Problem 2.1) that only those light rays that are incident at an angle

$$\theta_0 < \theta_0^{\max} = \sin^{-1} \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \tag{2.2}$$

at the air-core interface will undergo total internal reflection at the core-cladding interface and will thus propagate. Such rays are called *guided rays,* and $\theta_0^{\max}$ is called the *acceptance angle*. The refractive index difference $n_1 - n_2$ is usually small, and it is convenient to denote the fractional refractive index difference $(n_1 - n_2)/n_1$ by $\Delta$. For small $\Delta$, $\theta_0^{\max} \approx \sin^{-1} \frac{n_1\sqrt{2\Delta}}{n_0}$. As an example, if $\Delta = 0.01$, which is a typical value for (multimode) fiber, and $n_1 = 1.5$, a typical value for silica, assuming we are coupling from air, so that $n_0 = 1$, we obtain $\theta_0^{\max} \approx 12°$.

### 2.2.2 Bit Rate–Distance Limitation

Owing to the different lengths of the paths taken by different guided rays, the energy in a narrow (in time) pulse at the input of the fiber will be spread out over a larger time interval at the output of the fiber. A measure of this time spread, which is called *intermodal dispersion*, is obtained by taking the difference in time, $\delta T$, between the fastest and the slowest guided rays. Later we will see that by suitably designing the fiber, intermodal dispersion can be significantly reduced (graded-index fiber) and even eliminated (single-mode fiber).

We now derive an approximate measure of the time spread due to intermodal dispersion. Consider a fiber of length $L$. The fastest guided ray is the one that travels along the center of the core and takes a time $T_f = Ln_1/c$ to traverse the fiber, $c$ being the speed of light in a vacuum. The slowest guided ray is incident at the critical angle

on the core-cladding interface, and it can be shown that it takes a time $T_s = Ln_1^2/cn_2$ to propagate through the fiber. Thus

$$\delta T = T_s - T_f = \frac{L}{c}\frac{n_1^2}{n_2}\Delta.$$

How large can $\delta T$ be before it begins to matter? That depends on the bit rate used. A rough measure of the delay variation $\delta T$ that can be tolerated at a bit rate of $B$ b/s is half the bit period $1/2B$ s. Thus intermodal dispersion sets the following limit:

$$\delta T = \frac{L}{c}\frac{n_1^2}{n_2}\Delta < \frac{1}{2B}. \tag{2.3}$$

The capacity of an optical communication system is frequently measured in terms of the *bit rate–distance product*. If a system is capable of transmitting $x$ Mb/s over a distance of $y$ km, it is said to have a bit rate–distance product of $xy$ (Mb/s)-km. The reason for doing this is that usually the same system is capable of transmitting $x'$ Mb/s over $y'$ km providing $x'y' < xy$; thus only the product of the bit rate and the distance is constrained. (This is true for simple systems that are limited by loss and/or intermodal dispersion, but is no longer true for systems that are limited by chromatic dispersion and nonlinear effects in the fiber.) From (2.3), the intermodal dispersion constrains the bit rate–distance product of an optical communication link to

$$BL < \frac{1}{2}\frac{n_2}{n_1^2}\frac{c}{\Delta}.$$

For example, if $\Delta = 0.01$ and $n_1 = 1.5(\approx n_2)$, we get $BL < 10$ (Mb/s)-km. This limit is plotted in Figure 2.6.

Note that $\theta_0^{\max}$ increases with increasing $\Delta$, which causes the limit on the bit rate–distance product to decrease with increasing $\Delta$. The value of $\Delta$ is typically chosen to be less than 1% so as to minimize the effects of intermodal dispersion, and since $\theta_0^{\max}$ is consequently small, lenses or other suitable mechanisms are used to couple light into the fiber.

### 2.2.3 Controlling Intermodal Dispersion: Graded-Index Multimode Fiber

Thus far, we have assumed that the fiber is a *step-index* fiber since the variation of the refractive index along the fiber cross section can be represented as a function with a step at the core-cladding interface. In practice, however, multimode fibers have more

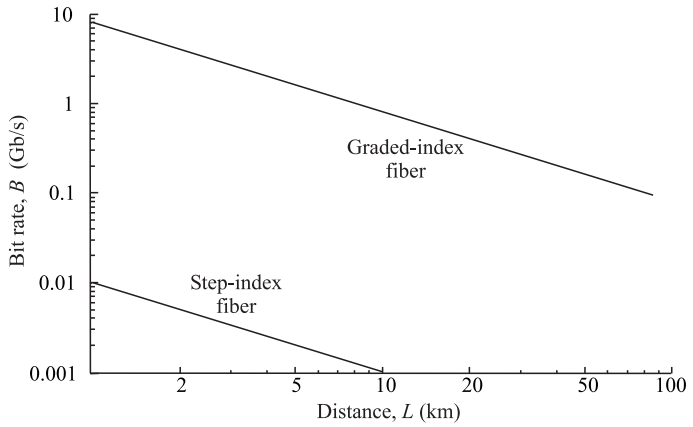


**Figure 2.6** Limit on the bit rate–distance product due to intermodal dispersion in a step-index and a graded-index fiber. In both cases, $\Delta = 0.01$ and $n_1 = 1.5$.

sophisticated *graded-index* profiles designed to reduce the intermodal dispersion. The refractive index decreases gradually, or continuously, from its maximum value at the center of the core to the value in the cladding at the core-cladding interface.

This has the effect of reducing $\delta T$ because the rays traversing the shortest path through the center of the core encounter the highest refractive index and travel slowest, whereas rays traversing longer paths encounter regions of lower refractive index and travel faster. For the optimum graded-index profile (which is very nearly a quadratic decrease of the refractive index in the core from its maximum value at the center to its value in the cladding), it can be shown that $\delta T$, the time difference between the fastest and slowest rays to travel a length $L$ of the fiber, is given by

$$\delta T = \frac{L}{c}\frac{n_1\Delta^2}{8}.$$

Assuming that the condition $\delta T < 1/2B$, where $B$ is the bit rate, must be satisfied, we get the following limit on the bit rate–distance product of a communication system employing graded-index fiber:

$$BL < \frac{4c}{n_1\Delta^2}.$$

For example, if $\Delta = 0.01$ and $n_1 = 1.5$, we get $BL < 8$ (Gb/s)-km. This limit is also plotted in Figure 2.6 along with the limit for step-index fiber.

**Table 2.1** A comparison of multimode fiber. Effective modal bandwidth (EMB) and overfilled launch bandwidth (OFLBW) correspond to laser and LED sources, respectively.

| Fiber Type | Also Known As | Core Diameter (Microns) | EMB–Distance at 850 nm (MHz-km) | OFLBW–Distance at 850/1300 nm (MHz-km) |
|---|---|---|---|---|
| OM1 | FDDI Grade | 62.5 | *Not Applicable* | 200/500 |
| OM2 | | 50.0 | *Not Applicable* | 500/500 |
| OM3 | Laser Optimized | 50.0 | 2000 | 1500/500 |
| OM4 | Laser Optimized | 50.0 | 4700 | 3500/500 |

### 2.2.4 *Multimode Fiber in Practice*

Since the bit rate-distance product is the limitation, each doubling of the bit rate will result in a halving of the transmission reach. Multimode fiber continues to be used heavily in data centers and corporate local-area networks (LANs). Much effort has been put into engineering successively higher speed transmitters and receivers to work over the installed base of older multimode fibers while maintaining the same maximum distance.

The most commonly found versions of multimode fiber as designated by the International Standards Organization (ISO) are shown in Table 2.1. They represent successive generations of products. OM1 was widely installed in the mid-1990s, and OM3 is the recommended installation today. (Note: other variations such as 50 micron OM1 are allowed by the standards.) Also shown is OM4 fiber, which at the time of this writing is expected to be standardized in 2009.

The bandwidth–distances, similar to bit rate–distances, are shown in the table. As the bandwidth is limited by differences in propagation between modes, the actual effective bandwidth in practice depends on what modes are launched. This largely depends on whether transmitters are light emitting diodes (LEDs) or lasers, as we explain next.

The multimode fiber technology roadmap is intertwined with transmitter technology progress. In the mid-1990s, multimode fiber was primarily for use with LED transmitters at 850 nm or 1300 nm. In order to specify the fiber, the bandwidth is measured under a controlled *overfilled launch* (OFL), which is essentially a uniform excitation of all modes in the fiber. This represents the emission characteristics of LEDs.

LEDs were limited to bit rates of 622 Mb/s and below. The majority of links today are 1 Gb/s to 10 Gb/s, using laser transmitters, with bit rates of 40 Gb/s and 100 Gb/s

planned for the future. An important type of laser used for these applications is the vertical cavity surface-emitting laser (VCSEL; see Subsection 3.5.1). These lasers transmit at 850 nm and up to 10 Gb/s. VCSELs at 1300 nm have been demonstrated but are not in wide use commercially. A different launch condition, called the *effective laser launch*, in which only a small subset of modes are excited, better represents what happens when laser transmitters are used. This launch condition is used to specify OM3 and OM4 fibers, where the bandwidth is referred to as the *effective modal bandwidth* (EMB). The uses of lasers instead of LEDs results in a higher bandwidth–distance product, as can be seen from Table 2.1. In practice, variations in alignment between laser and fiber, and variations in the lasers themselves, cause different modes to be excited, complicating the assurance of effective bandwidth. The solution has been to improve the quality of OM3 and OM4 fiber, particularly near the center of core, such that the dispersion is kept sufficiently small regardless of which subset of modes are excited.

### Parallel Ribbon Fiber

Although most fiber links are serial high-speed (and unidirectional) connections, there do exist parallel fiber connections as well. A typical parallel connection uses a standard 12-wide ribbon of fibers for a distance up to tens or hundreds of meters. The fibers are individually protected by a plastic jacket layer before being assembled side by side, where the spacing is 250 microns. The most common usage is multimode, where VCSEL arrays can be used as transmitters. Single-mode ribbon fiber exists as well. Connection specifications for parallel ribbon fiber are developed through multisource agreements (MSAs) among vendors. Example MSAs are SNAP12 and QSFP (Quad Small Form Factor Pluggable).

## 2.3 Optical Fiber as a Waveguide

In order to completely overcome intermodal dispersion, you must use fibers whose core radius is appreciably smaller and of the order of the operating wavelength. Such fibers have only one mode, ray, or path in which light can propagate. These fibers are called *single-mode* fibers.

A useful way to conceptualize propagation in a single-mode fiber is to treat the light as a single beam. The following physical explanation for the propagation of light in single-mode fiber is based on [Neu88]. In any medium with a constant refractive index, a narrow light beam tends to spread due to a phenomenon called *diffraction*. Thus, in such a medium, the beam width will increase as light propagates. Note that this diffraction phenomenon is what makes the geometric optical approach invalid for single-mode fibers, that is, an optical signal cannot be modeled as a ray.