where $\mathbf{I}$ is the identity matrix. Note that this relation follows merely from conservation of energy and can be readily generalized to a device with an arbitrary number of inputs and outputs.

For a 2×2 directional coupler, by the symmetry of the device, we can set $s_{21} = s_{12} = a$ and $s_{22} = s_{11} = b$. Applying (3.4) to this simplified scattering matrix, we get

$$|a|^2 + |b|^2 = 1 \tag{3.5}$$

and

$$ab^* + ba^* = 0. \tag{3.6}$$

From (3.5), we can write

$$|a| = \cos(x) \text{ and } |b| = \sin(x). \tag{3.7}$$

If we write $a = \cos(x)e^{i\phi_a}$ and $b = \sin(x)e^{i\phi_b}$, (3.6) yields

$$\cos(\phi_a - \phi_b) = 0. \tag{3.8}$$

Thus ϕ_a and ϕ_b must differ by an odd multiple of $\pi/2$. The general form of (3.1) now follows from (3.7) and (3.8).

The conservation of energy has some important consequences for the kinds of optical components that we can build. First, note that for a 3 dB coupler, though the electric fields at the two outputs have the same magnitude, they have a relative phase shift of $\pi/2$. This relative phase shift, which follows from the conservation of energy as we just saw, plays a crucial role in the design of devices such as the Mach-Zehnder interferometer that we will study in Section 3.3.7.

Another consequence of the conservation of energy is that *lossless combining* is not possible. Thus we cannot design a device with three ports where the power input at two of the ports is completely delivered to the third port. This result is demonstrated in Problem 3.2.

3.2 Isolators and Circulators

Couplers and most other passive optical devices are *reciprocal* devices in that the devices work exactly the same way if their inputs and outputs are reversed. However, in many systems there is a need for a passive *nonreciprocal* device. An *isolator* is an example of such a device. Its main function is to allow transmission in one direction through it but block all transmission in the other direction. Isolators are used in systems at the output of optical amplifiers and lasers primarily to prevent reflections from entering these devices, which would otherwise degrade their performance. The

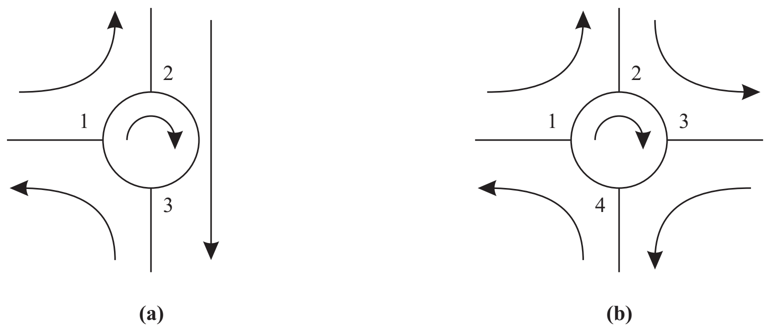

Figure 3.3 Functional representation of circulators: (a) three-port and (b) four-port. The arrows represent the direction of signal flow.

two key parameters of an isolator are its *insertion loss*, which is the loss in the forward direction and which should be as small as possible, and its *isolation*, which is the loss in the reverse direction and which should be as large as possible. The typical insertion loss is around 1 dB, and the isolation is around 40–50 dB.

A *circulator* is similar to an isolator, except that it has multiple ports, typically three or four, as shown in Figure 3.3. In a three-port circulator, an input signal on port 1 is sent out on port 2, an input signal on port 2 is sent out on port 3, and an input signal on port 3 is sent out on port 1. Circulators are useful to construct optical add/drop elements, as we will see in Section 3.3.4. Circulators operate on the same principles as isolators; therefore we only describe the details of how isolators work next.

3.2.1 Principle of Operation

In order to understand the operation of an isolator, we need to understand the notion of *polarization*. Recall from Section 2.3.3 that the *state of polarization* (SOP) of light propagating in a single-mode fiber refers to the orientation of its electric field vector on a plane that is orthogonal to its direction of propagation. At any time, the electric field vector can be expressed as a linear combination of the two orthogonal linear polarizations supported by the fiber. We will call these two polarization modes the horizontal and vertical modes.

The principle of operation of an isolator is shown in Figure 3.4. Assume that the input light signal has the vertical SOP shown in the figure. It is passed through a *polarizer*, which passes only light energy in the vertical SOP and blocks light energy in the horizontal SOP. Such polarizers can be realized using crystals, called *dichroics*,

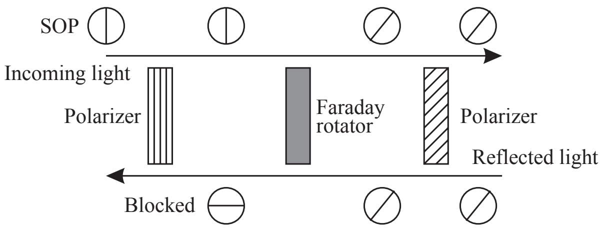

Figure 3.4 Principle of operation of an isolator that works only for a particular state of polarization of the input signal.

which have the property of selectively absorbing light with one SOP. The polarizer is followed by a *Faraday rotator*. A Faraday rotator is a nonreciprocal device, made of a crystal that rotates the SOP, say, clockwise, by 45°, regardless of the direction of propagation. The Faraday rotator is followed by another polarizer that passes only SOPs with this 45° orientation. Thus the light signal from left to right is passed through the device without any loss. On the other hand, light entering the device from the right due to a reflection, with the same 45° SOP orientation, is rotated another 45° by the Faraday rotator, and thus blocked by the first polarizer.

Note that the preceding explanation assumes a particular SOP for the input light signal. In practice we cannot control the SOP of the input, and so the isolator must work regardless of the input SOP. This requires a more complicated design, and many different designs exist. One such design for a miniature polarization-independent isolator is shown in Figure 3.5. The input signal with an arbitrary SOP is first sent through a *spatial walk-off polarizer* (SWP). The SWP splits the signal into its two orthogonally polarized components. Such an SWP can be realized using *birefringent* crystals whose refractive index is different for the two components. When light with an arbitrary SOP is incident on such a crystal, the two orthogonally polarized components are refracted at different angles. Each component goes through a Faraday rotator, which rotates the SOPs by 45°. The Faraday rotator is followed by a *half-wave plate*. The half-wave plate (a reciprocal device) rotates the SOPs by 45° in the clockwise direction for signals propagating from left to right, and by 45° in the counterclockwise direction for signals propagating from right to left. Therefore, the combination of the Faraday rotator and the half-wave plate converts the horizontal polarization into a vertical polarization and vice versa, and the two signals are combined by another SWP at the output. For reflected signals in the reverse direction, the half-wave plate and Faraday rotator cancel each other's effects, and the SOPs remain unchanged as they pass through these two devices and are thus not recombined by the SWP at the input.

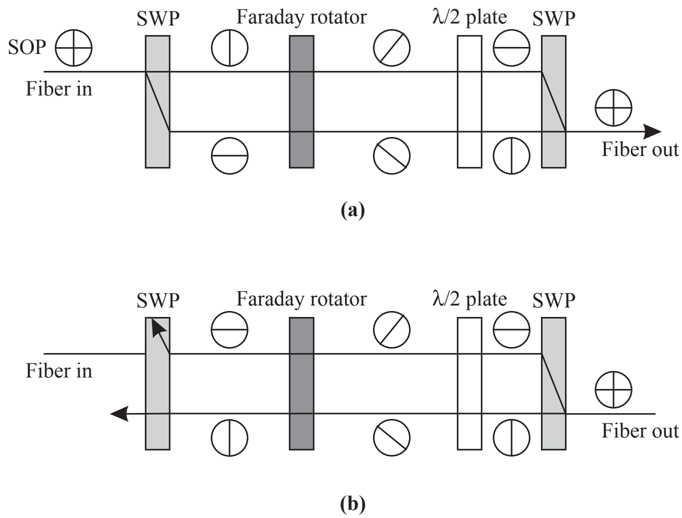

Figure 3.5 A polarization-independent isolator. The isolator is constructed along the same lines as a polarization-dependent isolator but uses spatial walk-off polarizers at the inputs and outputs. (a) Propagation from left to right. (b) Propagation from right to left.

3.3 *Multiplexers and Filters*

In this section, we will study the principles underlying the operation of a variety of wavelength selection technologies. Optical filters are essential components in transmission systems for at least two applications: to multiplex and demultiplex wavelengths in a WDM system—these devices are called multiplexers/demultiplexers—and to provide equalization of the gain and filtering of noise in optical amplifiers. Furthermore, understanding optical filtering is essential to understanding the operation of lasers later in this chapter.

The different applications of optical filters are shown in Figure 3.6. A simple filter is a two-port device that selects one wavelength and rejects all others. It may have an additional third port on which the rejected wavelengths can be obtained. A multiplexer combines signals at different wavelengths on its input ports onto a common output port, and a demultiplexer performs the opposite function. Multiplexers and demultiplexers are used in WDM terminals as well as in larger *wavelength crossconnects* and *wavelength add/drop multiplexers*.

Demultiplexers and multiplexers can be cascaded to realize *static* wavelength crossconnects (WXCs). In a static WXC, the crossconnect pattern is fixed at the time